If the Hessian $\nabla^2 f$ is positive (semi)definite then f is a convex function

Proof: Let $x, u \in \Omega$, Ω is a convex domain

By Taylor expansion around u,

$$f(x) = f(u) + \nabla f(u)(x-u) + \frac{1}{2}(x-u)^T \nabla^2 f(u^*)(x-u)$$

u^* is some point in Ω

Since the Hessian matrix $\nabla^2 f(v)$ is positive (semi)definite for all v in Ω, the last quadratic term (error term) in the above equation is non-negative

$$f(x) \geqslant f(u) + \nabla f(u)(x-u)$$

Now consider $x, y \in \Omega$ and any point z on the segment between x and y. We may express z by $tx + (1-t)y$, $t \in [0,1]$, and hence $z \in \Omega$.

Using the above inequality with the Taylor expansions around z

$$t\, f(x) \geqslant t\, f(z) + t \nabla f(z)(x-z)$$
$$(1-t) f(y) \geqslant (1-t) f(z) + (1-t) \nabla f(z)(y-z)$$

$\Rightarrow$

$$t f(x) + (1-t) f(y) \geqslant f(z) + \nabla f(z)\left(tx + (1-t)y - z\right)$$
$$= f(z)$$
$$= f(tx + (1-t)y)$$

Therefore $f(\cdot)$ is a convex function.